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Genetic Algorithm Application to Portfolio Optimisation

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Abstract

This paper presents an evolutionary algorithm (EA) capable of calculating the efficient frontier for a given portfolio. The objective of this paper is threefold. First of all, it is shown how the EA can be used to maximise the return of a portfolio while also minimising the risk. Secondly, the algorithm is used to find the global minimum for a variance optimisation problem with short-selling constraints. Lastly, analysis is carried out to determine which genetic operators are more relevant for the portfolio's optimisation problem and initial parameters of the EA, specifically population size and mutation rate, are tuned to optimise algorithm's solution and performance.

1 Introduction and motivation

Modern portfolio theory is based on Harry Markowitz's work on mean-variance portfolios [1], which stated that a rational investor should either maximise their expected return for a given level of risk, or minimise their risk for a given return. These two principles lead to an efficient frontier of portfolios, among which the investor is free to choose.

More than sixty years on, there is no widely accepted practical implementation of the mean-variance portfolio theory. Typical optimisers rely on quadratic programming and deterministic algorithms to find "optimal" portfolios. However, when practical limitations of the financial markets, such as transaction costs and minimal transaction lots, are included in the model it becomes an NP-hard problem and a global optimal solution cannot be obtained by traditional mathematics programming techniques.

In this paper an evolutionary algorithm (EA), which allows for much more freedom in the functional form, is used. These types of algorithms are heuristic and stochastic search methods, and are often well-suited to find good solutions to optimisation problems where the search space has many local minima and/or there are no known well-performing deterministic search methods. The proposed solution with no short-selling constraint has been extensively tested on several portfolios ranging in size from a few to hundreds of stocks. Please note that if the no short-selling constraint was to be relaxed, the EA would be more complex, since additional constraints would need to be introduced, such as maximum leverage level, maximum short position size, maximum proportion of assets to be held short and etc. Please see [2] for the detailed discussion of the impact of the short-selling constraint on the portfolio optimisation.

The rest of the paper is organised as follows. The next section sets out the description of the problem to be solved. Section 3 illustrates the EA which has been implemented, together with the chromosomes' representation, fitness functions and genetic operators used. Section 4 provides details of the parameter settings for the various experiments carried out with the proposed method. Section 5 discusses the experimental results of the proposed method. Conclusions are drawn in Section 6.

2 Problem description

From Markowitz's portfolio theory [1] it is known that with a portfolio of risky assets one can achieve an efficient portfolio (also called mean-variance efficient portfolio) that has the lowest level of risk for a given rate of the expected return. The goal here is to find the combination of assets that provides the following:

- the minimum variance portfolio, i.e. the portfolio on the efficient frontier with the lowest level of risk overall;
- the maximum Sharpe ratio.

2.1 Sharpe ratio

The Sharpe ratio, S, is a measure for quantifying the performance of the portfolio and is computed as follows:

$$S = \frac{\mu - r}{\sigma}$$

Here μ is the return of the portfolio over a specified period, *r* is the risk-free rate over the same period and σ is the standard deviation of the returns over the specified period. A 10 year treasury rate was used as the risk free rate in this paper. The next step would be to calculate the mean return and the volatility for the portfolio.

2.2 Holding period return

Suppose that portfolio consists of *N* risky assets. Then weight, w_i, with $i = 1 \ldots N$, can be defined as the percentage of the initial wealth invested in the asset i. The wealth amount invested in the asset *i* can be expressed as

$$a_i = w_i \cdot I,$$

where *I* represents the initial investment amount, such that $\sum_{i=1}^{N} a_i = I$. Since there is a constraint of not short-selling any stocks, the following holds:

$$\sum_{i=1}^{N} w_i = 1 \text{ and } \forall w_i \geq 0. \tag{1}$$

Therefore, for any given observation day, *d*, with *d = 0* as the reference day of calculations, the portfolio's value can be expressed as follows:

$$V_d = \sum_{i=1}^{N} \left(\frac{a_i}{p_{0i}} \cdot p_{di} \right) \quad \forall d \geq 0,$$

where p_{0i} is the price of the asset i at *d = 0* and p_{di} is the price of the asset *i* on the day *d*. The portfolio's return and the portfolio's volatility are obtained by calculating the portfolio's value for each trading day in the time horizon set for the simulation.

3 Evolutionary algorithm

Evolutionary algorithms (EA) are adaptive heuristic search and optimisation techniques, inspired by Charles Darwin's theory of natural evolution - the survival of the fittest individuals (see [3] and [4]), and have several practical applications.

A typical EA is based on the iterative process with the following steps:

- random generation of an initial population of possible solutions;

- evaluation of the fitness value of all individuals of the generated population, where fitness is the objective function of the problem to solve and the fitness value represents how good the solution is;
- selection of some individuals, where in most cases the fittest individuals are selected;
- generation of a new population using the selected individuals and by modifying and/or recombining the selected individuals' genes.

The next sections describe the EA for finding the optimal portfolio weights that maximise the Sharpe ratio and for finding the minimum variance portfolio. Please refer to [3] and [4] for more details. Specifics of the fitness function, genetic operators and chromosome representation are also provided.

3.1 Description of the algorithm implemented

The EA used for the simulation in this paper is the (μ + λ) evolution strategy, aka (μ + λ)-ES, introduced in [5], where in each generation λ worst individuals out of the total $\mu + \lambda$ are discarded. As a first step, $\mu + \lambda$ chromosomes of the initial population are randomly initialised. A chromosome (also called genotype) is a set of genetic material that fully describes an individual. Chromosome representation applicable to the problem investigated in this paper is discussed in Section 3.2. As a second step, the fitness value of each individual chromosome is evaluated. As the next step, the fittest μ individuals are selected from the population. In the last step, the best individual, i.e. the chromosome with the highest fitness value, is used to test if the conditions for stopping the evolutionary process have been met. These conditions are defined as follows: the fitness value, based on the best chromosome, has converged and/or the number of generations has reached the maximum value set for the simulation. If the conditions are not met, then a new population is generated and the four steps, described above, are repeated.

The generation of a new population is based on mating the individuals, selected in the third step of the algorithm, using genetic operators. The most common genetic operators are mutation, crossover and elitism. A small percentage of the new population is also generated using random chromosomes to bring more diversification into the gene pool.

3.2 Chromosome representation

In EA, a chromosome is a set of parameters that represent a solution to a given problem. Each such parameter is called gene (also allele) and it is the basic unit of information within the evolutionary process. For the optimisation problems considered in this paper, the chromosome represents the weights of the assets in the portfolio. If the portfolio is made up of *N* risky assets, then the chromosome is represented by an array of *N* positive real numbers, whose sum is 1, each of which represents a percentage of wealth. Let **w** be a chromosome, then

$$\mathbf{w} = \{w_1, w_2, \cdots, w_{N-1}, w_N\}$$

Each gene, w_i, of the chromosome represents the wealth invested in an asset, i, and it must be positive – short selling is not allowed.

3.3 Fitness function

For each round of population generation, the fitness value is evaluated for each chromosome in the population. In other words, the fitness function evaluates the evolved portfolio of weights. For example, if the task is to find the minimum variance portfolio, then the fitness value is the standard deviation of the portfolio and this would be minimised for the best solution. If, on the other hand, the task is to find the largest Sharpe ratio, then the fitness value is the Sharpe ratio for the portfolio calculated as if the asset's allocation is as described by the chromosome.

3.4 Genetic operators

Crossover, elitism, gene mutation and new random genes, referred to here as min max operator, have been used in this paper to generate the new population. Crossover combines the genes of two parents to generate one or more offspring. Crossover causes the exchange of genetic materials between the two parents with the possibility that the new genes generated this way are bfitter (e.g. have higher

Parent A	5.18	75.03	0.75	1.40	8.37	7.39	0.82	1.06
Parent B	5.97	75.92	5.70	2.75	1.52	6.65	0.61	0.88
						crossover point		
Offspring A	5.18	75.03	0.75	1.40	8.37	7.39	0.61	0.88
Offspring B	5.97	75.92	5.70	2.75	1.52	6.65	0.82	1.06

Figure 1: Single Point Crossover

fitness function value for the maximisation problem) than their parents. It can significantly speed up the evolutionary process in terms of number of generations required to find the optimal solution. Elitism ensures that the fittest individuals of one generation are carried on to the next. Mutation operator involves changing the genes of the chromosome. These genetic operators are discussed in more detail in the next sections.

3.4.1 Elitism operator

Elitism takes the fittest individuals of the current generation and uses them as they are in the next generation. In the (μ + λ) evolutionary strategy μ is the number of individuals copied to the next generation, and for the numerical experiments in this paper $\mu = 10$ is used unless stated otherwise.

3.4.2 Crossover operator

The crossover operators used in this paper are the single point crossover, the flat crossover [6] and the blend crossover [7]. The single point crossover randomly selects a crossover point, then the parents are cut at the crossover point (randomly selected) and subsequently recombined with a piece of each other to generate two descendants. The offspring are then normalised to satisfy equation 1 in Section 2.2. Figure 1 provides an example of the single point crossover method.

The flat crossover, introduced in [8], generates offspring from parents by selecting a random number from the range and using a linear combination of parents' genes, such that child's gene, i, is computed as follows:

$$w_{i,child} = \lambda_i \cdot w_{i,parent_A} + (1 - \lambda_i) \cdot w_{i,parent_B}$$

For the standard flat crossover λ is a uniform random number between 0 and 1. In this paper's experiments the value of flat crossover lambda set to 0.7 has produced the best results.

The blend crossover (BLX-α) was introduced in [7] as a generalisation of the flat crossover. The blend crossover generates offspring by uniformly picking values within an interval that contains the two parents. The lower and upper bound of the interval are calculated from the parent's genes as follows:

$$\begin{aligned} \text{UpperBound}_i &= \max(w_{i,parent_A}, w_{i,parent_B}) + \alpha \cdot d_i \\ \text{LowerBound}_i &= \min(w_{i,parent_A}, w_{i,parent_B}) - \alpha \cdot d_i \\ d_i &= \mid w_{i,parent_A} - w_{i,parent_B} \mid \end{aligned}$$

where the coefficient, α, is selected within the interval [0, 1]. Several simulations with different portfolios and different time horizons have been run in order to find the best value for the coefficient. Results of these experiments are described in Section 4.2.

3.4.3 Mutation operator

The mutation operator, see [3] for more details, modifies one or more genes from the parent's chromosome to generate one or more new chromosomes. However, because the sum of the weights in the chromosome must be one, all the other genes are normalised. Two different mutation operators have been used in this paper. These can be referred to as Genes mutation (GM-μ) and Bump mutation (BM-μ). These mutation operators are applied to the fittest parents' chromosomes, as previously described in Section 3.1

In GM-μ, for each parent, two offspring are generated in the following way:

- The largest gene (weight) is increased and the smallest gene is decreased by a factor μ;

- The largest gene (weight) is decreased and the smallest gene is increased by a factor μ.

In BM-μ, for each parent, two offspring are generated by modifying a percentage of the parent's chromosome. A percentage, x, of the parent's chromosome is randomly selected, then half of those genes are increased and half are decreased by a factor μ.

The factor μ of the GM-μ and BM-μ can be selected by the user, and its value is between 0 and 1. Section 4.3 shows the impact of each of these operators.

3.4.4 Min Max operator

A small percentage of new random genes are produced for each population generation. However, the new weights are not randomly allocated to the portfolio's assets. Instead, the following rule is applied: the largest weight is assigned to the asset which already had the largest weight in the previous generation and the smallest to the smallest weight, and so on.

4 Experiments and Results

In this section the results of the evolution of the portfolio's weights together with the tuning of the parameters are examined. The objectives of the experiments are:

- to select the best genetic operators, together with the tuned input parameters for the type of problem considered in this paper; and
- to show that the $(\mu + \lambda)$-ES is capable of optimising a portfolio of risky assets in a relatively short time frame (in terms of number of generations required).

Figure 2 shows the evolution for maximising the Sharpe ratio for a portfolio of 14 assets. It can be noticed how the Sharpe ratio increases generation after generation especially for the first few generations. In the graph, the stalling effect between generation 23 and 32 can be observed, after which the fitness value keeps increasing until it finds the optimal value for the Sharpe ratio. The figure provides values for both the fittest individual (blue) of the population as well as the average fitness values of the entire population (orange).

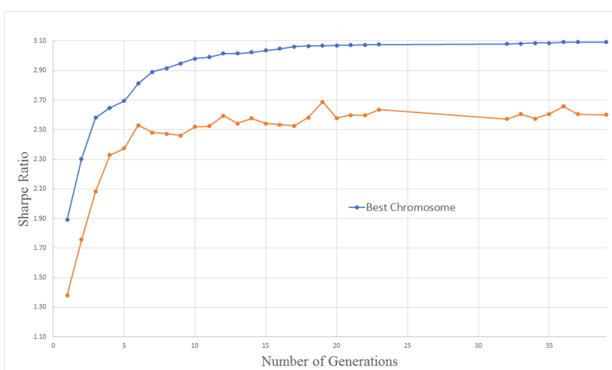

Figure 2: Sharpe Ratio vs number of generations for the fittest individual (blue) and the population average fitness values (orange)

Although, the main focus of this paper is tuning of parameters in order to find the minimum variance and the largest Sharpe ratio for a given choice of assets, the current implementation of the proposed evolution strategy can also be used to obtain the efficient frontier of the portfolio. Evolution of the chromosomes generation after generation is shown together with the efficient frontier in Figure 3. A quick clustering effect can be observed, i.e. grouping of the third generation (Generation 3) circled in black compared against the spread out pattern of the initial generation (Generation 1).

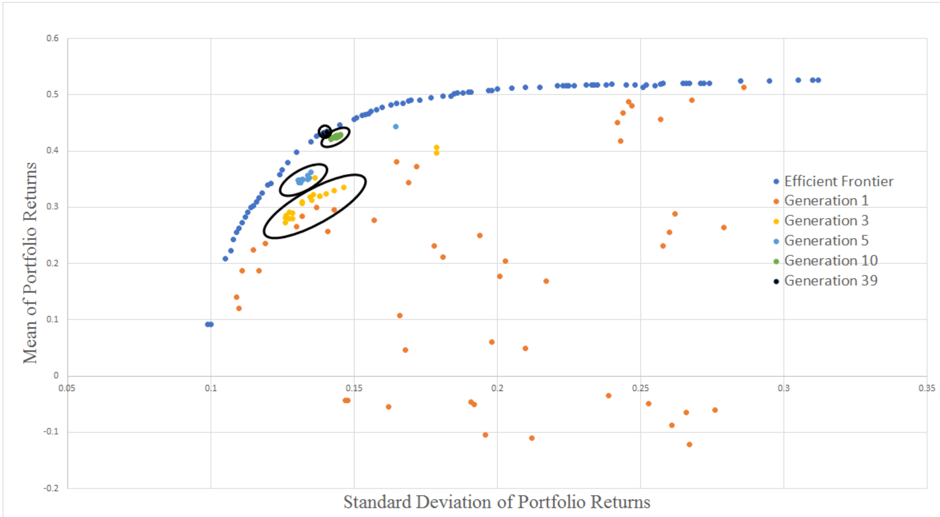

Figure 3: Efficient Frontier generated using the proposed evolutionary algorithm

4.1 Dataset

Several experiments have been undertaken with portfolios of different sizes and over different time periods. The experiments have been carried out using historical prices ranging across a period of 10 years. The adjusted prices, taken from Yahoo Finance, have been used.

For the experiments, the stocks for the portfolio were randomly chosen from the following list: A, AA, AAP, AAPL, ABC, ABT, ACN, ADBE, ADI, ADM, ADP, ADS, ADSK, AEP,AES, AET, AFL, AGN, AIG, AIV, AIZ, AKAM, ALXN, AMAT, AME, AMG, AMGN, AMT, AMZN, AN, ANTM, AON, APA, APC, APD, APH, AVB, AVY, AXP, AZO, BA, BAC, BAC, BAX,BBBY, BBT, BBY, BDX, BEN, BF-B, BK, BLL, BLX, BMY, BRK-B, BWA, BXP, C, CA, CAG, CAH, CAT, CB, CCE, CCI, CCL, CELG, CERN, CHRW, CI, CINF, CL, CLX, CMA, CMCSA, CME, CMI, CMS, CNP, CNX, COF, COG, COL, COP, COST, CPB, CRM, CSCO, CSX, CTAS, CTL, CTSH, CTXS, CVS, CVX, D, DE, DHI, DHR, DIS, DLTR, DNB, DO, DOV, DRI, DTE, DUK, DVA, DVN, EA, EBAY, ECL, ED, EEM, EFX, EIX, EL, EMN, EMR, ENDP, EOG, EQIX, EQR, EQT, ES, ESRX, ESS, ESV, ETFC, ETN, ETR, EW, EXC, EXPD, F, FAST, FCX, FDX, FE, FFIV, FIS, FITB, FLIR, FLR, FLS, FMC, FOSL, FOXA, FTI, FTR, GD, GE, GILD, GIS, GLW, GME, GOOG, GOOGL, GPC, GRMN, GS, GT, GWW, HAL, HAS, HBAN, HCP, HD, HES, HIG, HOG, HON, HP, HPQ, HRB, HRL, HRS, HSIC, HST, HSY, HUM, IBM, IFF, INTC, INTU, IP, IPG, IR, IRM, ISRG, ITW, IVZ, JBHT, JCI, JEC, JNJ, JNJ, JNPR, JPM, JWN, K, KEY, KIM, KLAC, KMB, KMX, KO, KR, KSS, KSU, L, LB, LEG, LEN, LH, LLL, LLY, LM, LMT, LNC, LOW, LRCX, LUV, M, MAC, MAR, MAS, MAT, MCD, MCHP, MCK, MCO, MDLZ, MDT, MET, MHK, MKC, MLM, MMC, MMV, MNST, MO, MOS, MRK, MRO, MS, MSFT, MSI, MTB, MU, MUR, MYL, NBL, NDAQ, NE, NEE, NEM, NFLX, NFX, NI, NKE, NOC, NOV, NRG, NSC, NTAP, NTRS, NUE, NVDA, NWL, O, OI, OKE, OMC, ORCL, ORLY, OXY, PAYX, PBCT, PBI, PCAR, PCG, PDCO, PEG, PEP, PFE, PFG, PG, PGR, PH, PHM, PKI, PLD, PNC, PNR, PNW, PRGO, PRU, PSA, PVH, PWR, PX, PXD, QCOM, R, REGN, RF, RHI, RHT, RIG, RL, ROK, ROP, ROST, RRC, RTN, SBUX, SCG, SCHW, SEE, SHW, SJM, SLB, SLG, SNA, SO, SPG, SPY, SRCL, SRE, STI, STT, STX, STZ, SWK, SWN, SYK, SYMC, SYY, T, TAP, TGNA, TGT, THC, TIF, TMK, TMO, TROW, TRV, TSCO, TSN, TSS, TXN, TXT, UHS, UNH, UNM, UNP, UPS, URBN, URI, UTX, VAR, VFC, VLO, VMC, VNO, VRSN, VRTX, VTR, VZ, WAT, WBA, WDC, WEC, WFC, WHR, WM, WMB, WMT, WY, WYNN, XEC, XEL, XL, XLNX, XOM, XOM, XRAY, XRX, YUM, ZBH, ZION.

4.2 Analysis of the α coefficient in Blend Crossover

This section describes the results of the experiments carried out in order to find the optimal value of the α coefficient for the BLX(α) crossover. To undertake this analysis, the EA was modified such that only the BLX(α) operator was used and all other genetic operators had been removed.

1Year	Number of Assets in the portfolio							
Alpha	**4**	**6**	**8**	**10**	**12**	**14**	**16**	**18**
0.10	14.93	12.90	20.15	18.60	13.00	26.05	31.10	17.55
0.15	11.64	14.10	24.85	21.20	21.20	29.20	32.10	20.20
0.20	7.93	14.80	29.40	19.55	16.25	26.70	33.10	22.60
0.25	7.86	14.10	31.75	27.55	18.25	30.65	37.60	23.20
0.30	7.93	12.85	33.65	22.65	26.71	30.35	36.15	25.40
0.35	5.46	13.05	34.30	25.30	27.13	19.85	29.00	22.40
0.40	5.50	10.20	35.35	20.60	27.63	19.80	26.95	22.60
0.45	5.00	9.70	31.85	24.75	18.13	15.80	20.50	17.80
0.50	5.23	10.05	19.95	19.30	68.80	15.60	18.90	13.60
0.55	5.00	9.85	17.30	15.40	60.50	13.65	16.25	13.65
0.60	5.07	10.45	17.10	14.60	14.44	14.45	16.25	13.55
0.65	5.25	9.60	16.20	15.55	13.40	14.10	17.00	12.70
0.70	4.82	9.90	15.85	14.60	15.13	12.95	14.50	13.25
0.75	4.80	9.50	15.70	15.35	11.71	12.55	16.35	12.60
0.80	5.14	9.90	15.10	12.95	12.29	13.10	15.95	12.10
0.85	4.86	10.65	15.65	14.65	18.60	13.05	15.20	13.05
0.90	4.85	10.10	14.80	12.55	15.10	12.30	15.05	12.10
0.95	4.71	10.10	15.25	13.50	12.56	11.95	15.45	12.40

2 Years	Number of Assets in the portfolio							
Alpha	**4**	**6**	**8**	**10**	**12**	**14**	**16**	**18**
0.10	10.50	14.30	22.15	14.56	19.45	22.95	22.40	18.45
0.15	12.35	16.70	24.70	13.65	19.50	27.30	26.70	23.15
0.20	9.18	16.40	29.75	19.00	22.05	28.95	26.40	25.50
0.25	8.76	15.50	32.00	16.41	19.90	23.65	30.85	24.80
0.30	7.00	14.35	32.65	19.22	21.75	25.55	26.55	36.65
0.35	5.69	13.80	29.70	17.00	22.55	20.80	18.50	22.45
0.40	7.06	12.00	28.35	19.83	23.35	16.75	22.40	25.75
0.45	5.35	11.30	23.95	18.84	20.45	20.85	17.80	20.55
0.50	5.13	10.60	18.85	16.35	20.05	17.05	16.70	19.85
0.55	5.00	10.15	17.20	18.20	16.85	13.90	14.50	19.35
0.60	4.94	10.25	14.45	25.45	15.20	13.90	14.00	16.85
0.65	5.33	10.30	14.75	17.53	13.95	13.20	14.25	17.05
0.70	5.00	10.45	14.20	14.00	13.00	13.00	13.40	16.05
0.75	4.94	9.95	13.40	13.75	12.55	12.15	13.85	15.45
0.80	5.25	9.80	12.90	12.26	12.35	12.25	13.45	17.10
0.85	4.92	9.95	12.45	18.75	12.00	12.65	13.25	16.25
0.90	5.14	10.65	12.05	11.55	12.85	12.20	13.50	14.80
0.95	4.82	10.75	12.50	11.95	11.85	11.90	13.50	16.05

Table 1: Average number of generations to find the largest Sharpe ratio, by changing the α coefficient of the BLX-α operator for portfolios of different sizes. Two different time periods of one and two years have been tested. The best solution for each portfolio's size in terms of number of generations is highlighted in red.

1 Year	Number of Assets in the portfolio							
Alpha	**4**	**6**	**8**	**10**	**12**	**14**	**16**	**18**
0.10	7.80	9.80	20.95	16.70	16.45	22.50	23.05	24.55
0.15	8.55	10.10	23.15	18.55	18.85	23.95	25.75	28.55
0.20	7.55	9.70	26.90	20.10	20.20	27.80	30.55	32.55
0.25	7.75	9.50	30.75	23.95	22.85	33.15	35.55	36.90
0.30	8.85	9.25	37.40	26.10	26.10	36.40	41.60	40.85
0.35	7.00	8.60	39.55	29.60	27.60	43.25	36.45	45.70
0.40	6.40	9.10	43.30	32.85	30.65	60.10	27.90	32.15
0.45	7.35	8.70	35.95	28.00	25.40	43.10	24.00	25.55
0.50	6.50	8.80	28.00	24.30	22.80	27.00	20.45	22.10
0.55	6.80	8.70	24.75	21.35	20.40	24.00	18.00	21.10
0.60	6.70	9.20	23.05	20.00	19.10	20.80	18.05	20.30
0.65	6.80	9.30	21.60	19.85	19.55	20.00	18.00	20.95
0.70	6.10	9.35	21.40	20.30	19.95	20.10	18.35	20.35
0.75	6.75	9.90	22.30	20.35	20.10	19.65	17.60	20.30
0.80	6.60	9.55	21.75	20.70	20.95	19.95	19.35	20.65
0.85	5.65	9.60	21.00	22.25	20.70	21.15	19.05	20.45
0.90	6.95	9.65	21.70	21.95	21.75	21.75	19.75	21.85
0.95	6.20	10.75	22.00	24.70	23.70	22.35	20.25	22.10

2 Years	Number of Assets in the portfolio							
Alpha	**4**	**6**	**8**	**10**	**12**	**14**	**16**	**18**
0.10	6.45	7.95	19.95	15.05	15.50	20.40	22.25	23.60
0.15	6.70	7.35	21.60	17.65	17.85	22.85	24.45	27.95
0.20	7.05	8.00	25.55	19.05	19.95	24.60	28.20	30.10
0.25	6.15	7.95	29.70	22.40	22.60	29.10	32.10	36.80
0.30	6.65	7.80	33.75	23.35	24.80	31.75	36.55	41.15
0.35	6.20	8.10	37.15	25.70	27.85	30.80	37.35	42.15
0.40	5.85	8.10	37.60	23.70	27.40	31.40	29.35	32.50
0.45	6.55	8.25	30.30	22.35	25.00	31.30	22.65	24.50
0.50	5.50	8.25	24.15	20.95	22.70	23.35	20.85	22.15
0.55	18.65	8.70	21.30	19.30	18.95	19.60	19.00	21.10
0.60	6.40	8.05	20.05	19.55	19.80	19.25	18.50	21.30
0.65	6.75	8.30	19.50	18.20	19.40	18.70	17.60	20.90
0.70	6.60	9.30	18.90	19.35	19.15	18.50	18.45	21.50
0.75	6.95	9.00	19.75	19.80	19.90	19.40	19.05	21.50
0.80	6.32	9.20	19.30	19.00	20.30	20.35	18.25	22.40
0.85	6.40	9.15	19.50	20.65	21.05	21.15	20.00	23.05
0.90	6.35	9.80	20.45	22.30	21.55	21.65	21.00	23.20
0.95	6.05	10.40	20.75	23.40	24.45	22.25	20.30	25.65

Table 2: Average number of generations to find the minimum variance portfolio, by changing the α of the BLX-α operator for portfolios of different sizes. Two different time periods of one and two years have been tested. The best solution for each portfolio's size in terms of number of generations is highlighted in red.

Different size portfolios, number of assets ranging from 4 to 18, have been tested for values of $\alpha \in [0.10, 0.95]$ with the step size of 0.05. For each simulation the assets for each portfolio were randomly selected from the list in Table 4.1. The evolution strategy was run 100 times for each combination of inputs to obtain the average number of generations required to find either the largest Sharpe ratio or the minimum variance portfolios. It is worth noting, that when attempting to evolve larger portfolios (e.g. more than 18 assets), the algorithm was not able to find optimal results using only the blend crossover.

Results for the Sharpe ratio optimisation, presented in Table 1, demonstrate a clear pattern: for each portfolio the lowest number of generations is observed when the value of α is greater than or equal to 0.70. This conclusion is valid for both the 1 year and the 2 year time horizons. However, when considering the minimum variance of the portfolio instead, as shown in Table 2, no such pattern is evident.

4.3 Analysis of the mutation rate, μ, for the GM and BM operators

This section shows the results, in terms of number of generations required, for different values of the mutation rate, μ, used for the GM-μ and BM-μ mutation operators.

Max Sharpe - 1 year		Number of Assets in the portfolio								
Genetic Operator	μ	8	10	12	14	16	18	20	22	24
all	0.05	18.36	88.84	67.36	116.80	181.08	100.56	220.12	137.80	183.36
all	0.1	10.77	59.04	32.56	70.20	142.24	67.28	133.48	131.56	156.40
all	0.15	9.13	45.48	22.56	46.24	89.64	48.04	113.00	103.48	141.76
all	0.2	7.53	24.80	17.44	30.68	71.28	37.52	95.00	95.40	109.48
all	0.25	7.18	18.72	14.36	25.16	47.64	27.76	74.76	79.28	87.76
all	0.3	7.45	15.52	13.12	22.40	43.20	25.00	61.60	64.04	92.28
all	0.35	6.31	14.24	10.48	18.04	29.84	18.80	55.68	49.76	94.20
all	0.4	6.76	12.56	10.12	16.12	26.88	17.96	70.16	47.80	67.20
all	0.45	6.26	12.16	10.16	15.08	25.40	16.48	51.16	41.52	63.32
all	0.5	6.32	10.16	9.04	14.40	25.00	14.16	57.20	43.60	66.28
all	0.55	6.93	10.96	9.00	12.92	19.44	14.20	50.04	45.40	52.40
all	0.6	7.50	10.08	7.88	13.00	24.44	13.56	56.32	36.12	53.32
all	0.65	6.90	10.16	7.12	12.48	20.08	12.80	45.76	42.92	57.88
all	0.7	7.19	10.16	7.72	12.76	18.64	13.08	63.64	31.56	50.20
all	0.75	5.74	9.16	7.08	11.32	20.08	11.76	51.80	33.16	54.64
all	0.8	7.05	8.72	6.68	11.12	17.76	11.72	65.56	41.60	46.56
all	0.85	7.67	8.12	7.52	10.28	20.56	12.84	46.64	40.32	51.48
all	0.9	7.33	7.36	7.20	10.12	17.68	11.44	60.16	37.00	57.44
all	0.95	7.04	7.68	6.12	9.60	16.44	10.32	64.12	28.20	53.48

Max Sharpe - 2 years		Number of Assets in the portfolio								
Genetic Operator	μ	8	10	12	14	16	18	20	22	24
all	0.05	18.76	72.60	110.36	200.72	114.72	210.92	138.76	149.48	118.44
all	0.1	12.48	40.52	97.28	114.64	133.00	135.84	93.40	102.28	117.20
all	0.15	9.48	31.64	45.08	65.64	91.12	86.88	69.36	68.96	93.80
all	0.2	7.56	23.24	37.32	47.92	63.40	63.64	54.12	59.52	78.68
all	0.25	6.76	16.84	25.36	33.64	43.56	48.20	48.00	52.20	59.88
all	0.3	6.00	15.00	22.44	31.04	34.40	36.96	43.52	54.20	51.20
all	0.35	6.44	13.52	18.48	21.80	30.56	31.72	41.52	39.08	45.00
all	0.4	5.80	12.16	16.56	20.44	21.40	27.32	33.16	40.40	41.12
all	0.45	5.80	11.40	15.24	16.72	18.04	24.40	34.56	30.20	37.04
all	0.5	5.96	10.72	13.16	16.28	15.96	25.00	29.72	29.72	40.96
all	0.55	6.24	10.40	13.32	14.60	17.44	26.36	24.80	41.40	40.80
all	0.6	5.88	10.00	11.12	14.20	17.12	20.08	32.04	31.20	31.88
all	0.65	6.50	8.88	12.16	13.68	12.04	20.36	28.08	28.68	31.72
all	0.7	5.44	9.72	9.80	13.00	15.72	23.12	21.64	26.60	32.68
all	0.75	6.28	9.00	9.72	12.44	12.88	18.12	31.60	32.76	47.88
all	0.8	5.52	9.44	10.80	12.04	15.44	20.40	32.24	27.16	30.48
all	0.85	6.58	8.52	10.00	12.48	10.92	16.28	35.04	37.64	35.40
all	0.9	5.92	7.92	9.20	10.80	13.92	19.76	30.92	31.08	45.08
all	0.95	6.00	8.48	9.60	10.72	12.44	16.04	41.96	37.40	36.96

MinVol - 1 year		Number of Assets in the portfolio								
Genetic Operator	μ	8	10	12	14	16	18	20	22	24
all	0.05	8.12	107.40	181.12	197.96	268.60	323.56	294.00	334.60	349.28
all	0.1	6.28	66.00	107.08	124.44	163.44	199.60	186.84	225.68	244.96
all	0.15	5.32	47.32	76.16	94.04	110.88	130.32	131.68	163.60	174.20
all	0.2	5.56	37.76	62.72	74.00	83.64	116.92	101.40	134.40	150.12
all	0.25	5.12	30.32	54.12	63.32	71.96	85.28	76.72	108.08	102.88
all	0.3	5.12	27.40	36.56	50.80	60.76	74.84	67.44	86.44	93.52
all	0.35	5.24	27.44	35.60	51.20	57.40	65.04	66.64	77.56	86.88
all	0.4	5.28	22.72	35.28	42.68	51.60	53.52	63.20	70.00	79.04
all	0.45	5.48	18.56	24.80	36.68	45.44	55.48	60.68	64.60	70.40
all	0.5	5.24	22.08	24.40	35.72	39.80	46.56	50.44	59.40	71.16
all	0.55	5.28	15.88	27.88	31.16	34.32	39.88	49.08	59.36	61.92
all	0.6	5.47	15.60	25.88	25.76	40.00	44.68	46.64	54.24	55.16
all	0.65	5.42	16.00	21.64	24.92	36.76	36.72	42.92	56.36	58.16
all	0.7	5.59	13.40	15.32	23.64	29.48	32.48	46.12	53.96	57.20
all	0.75	5.60	14.84	20.84	20.80	31.12	38.48	41.20	46.64	49.56
all	0.8	5.53	14.08	17.16	22.64	28.56	31.12	34.24	41.16	48.28
all	0.85	5.33	14.16	19.80	24.44	22.80	40.16	31.24	45.52	47.40
all	0.9	5.50	12.16	20.20	18.56	29.00	29.96	37.28	38.08	50.80
all	0.95	5.17	12.80	17.84	19.32	22.56	26.08	39.04	40.60	41.60

MinVol - 2 years		Number of Assets in the portfolio								
Genetic Operator	μ	8	10	12	14	16	18	20	22	24
all	0.05	6.24	89.36	137.40	171.32	211.24	322.56	264.76	311.52	429.48
all	0.1	6.32	59.32	79.80	106.24	132.24	174.28	162.20	176.64	225.72
all	0.15	5.52	41.12	60.52	83.48	92.16	142.88	120.80	151.84	188.64
all	0.2	5.28	30.48	53.32	64.20	75.24	97.36	94.44	112.12	150.32
all	0.25	5.20	28.56	42.88	51.80	65.92	84.92	85.16	93.08	117.12
all	0.3	5.08	22.12	38.48	47.56	57.64	66.96	69.32	86.24	101.56
all	0.35	5.24	22.76	33.52	38.68	49.52	55.04	65.36	80.92	85.84
all	0.4	5.00	20.24	33.56	37.44	44.40	53.44	62.68	71.40	83.00
all	0.45	5.12	15.12	26.24	36.44	36.44	50.72	49.64	67.84	68.20
all	0.5	5.16	15.16	25.68	29.76	33.76	41.80	50.24	55.72	70.48
all	0.55	5.52	16.56	23.44	28.00	31.00	35.40	44.92	50.00	60.92
all	0.6	5.52	13.28	18.36	29.68	32.68	33.24	41.40	48.48	58.44
all	0.65	5.48	10.84	25.04	28.44	30.20	33.48	41.04	46.76	46.88
all	0.7	5.06	11.76	17.48	23.64	24.04	30.32	37.88	45.96	51.84
all	0.75	5.32	10.24	20.16	22.08	27.44	23.72	33.36	36.40	46.84
all	0.8	5.33	11.28	16.76	21.04	27.40	24.52	36.24	39.16	49.60
all	0.85	5.47	11.20	19.16	19.76	25.04	28.16	30.44	44.20	42.28
all	0.9	5.56	8.80	18.56	18.60	23.84	21.96	33.64	29.84	40.04
all	0.95	5.82	10.68	15.20	22.12	21.44	25.44	30.64	35.56	33.24

Table 3: Average of 100 runs of the number of generations for evolving portfolios of different sizes with different values of μ. All genetic operators described in section 3.4 were used in the evolution.

Different size portfolios, number of assets ranging from 8 to 24, have been tested for values of $\mu \in [0.05, 0.95]$ with the step size of 0.05. For each simulation the assets for each portfolio were randomly selected from the list in Table 4.1. The evolution strategy was run 100 times for each combination of inputs to obtain the average number of generations required to find either the largest Sharpe ratio or the minimum variance portfolios. Two sets of simulations were run – the first set included all genetic operators of the EA (Table 3), and the second set consisted of only the GM-μ and BM-μ (Table4).

It is worth noting, that when using all genetic operators, the optimum value for μ is above 0.70, with the exception of the smallest portfolio. However, When the EA is using only the GM-μ and BM-μ operators, no real pattern can be observed.

4.4 Comparison of different genetic operators

The objective of the experiments in this section is to identify the genetic operator that is the most relevant and plays the most significant role in arriving at the optimum solution. The genetic operators investigated here are the flat crossover, the blend crossover, the single point crossover, the gene mutation, the bump mutation and the min max operator. In order to carry out this analysis, the history of each chromosome since the first generation is tracked; i.e. all genetic operators, that have been applied to each single chromosome during the life of the evolutionary algorithm, are recorded. At the end of the simulation, analysis is done on how the chromosome, that solved the problem, has evolved and which operators have been applied to it since the first generation. For example, a hypothetical chromosome could have the following history (as shown in Table 5): at generation 1 the GM-μ operator was applied, at generation 2 the flat crossover was applied, and so forth. The application percentage for each of the operators is shown in the last row of the Table 5.

The final results are shown in Table 6. For these simulations, portfolios of 10 and 20 stocks have been selected. 100 simulations for each combination for the time period of 1, 2, 5 and 10 years and population sizes from 50 to 1000 individuals have been run.

Max Sharpe - 1 year		Number of Assets in the portfolio								
Genetic Operator	μ	8	10	12	14	16	18	20	22	24
GM-BM	0.05	21.11	27.00	67.42	39.36	33.50	27.92	33.91	14.00	23.80
GM-BM	0.1	17.11	9.29	32.83	21.82	31.00	36.91	10.10	12.00	17.20
GM-BM	0.15	12.00	10.38	44.27	27.82	12.67	31.50	8.44	10.00	14.11
GM-BM	0.2	11.20	10.50	35.08	33.50	46.45	15.36	6.89	10.55	22.36
GM-BM	0.25	11.78	11.29	28.54	23.82	41.50	18.70	14.11	21.30	14.90
GM-BM	0.3	10.40	12.43	37.60	19.09	33.91	26.91	7.00	10.64	11.30
GM-BM	0.35	11.57	9.14	26.50	37.91	25.17	12.64	8.40	19.10	17.50
GM-BM	0.4	11.44	7.67	20.77	31.33	32.55	25.64	6.78	12.20	12.89
GM-BM	0.45	10.00	8.80	37.46	26.60	34.55	20.36	12.00	9.50	9.90
GM-BM	0.5	12.20	8.14	24.33	23.17	20.27	14.70	7.56	9.90	28.80
GM-BM	0.55	11.00	6.13	33.45	20.36	22.62	30.00	7.50	8.60	10.36
GM-BM	0.6	12.14	8.83	31.73	24.45	25.92	12.09	10.90	11.70	38.18
GM-BM	0.65	10.50	10.33	24.27	26.90	24.42	17.42	8.44	10.89	22.00
GM-BM	0.7	14.13	9.50	18.90	33.27	26.00	23.00	7.11	11.00	28.44
GM-BM	0.75	11.27	11.50	28.27	14.10	13.55	21.36	6.56	8.55	21.80
GM-BM	0.8	10.75	6.67	21.09	21.82	33.92	29.17	6.63	17.20	9.40
GM-BM	0.85	14.67	7.22	38.42	43.64	33.77	39.40	9.80	9.78	10.22
GM-BM	0.9	12.29	8.83	17.33	21.55	18.54	15.00	7.11	7.88	13.22
GM-BM	0.95	11.75	9.50	20.82	25.33	34.27	19.27	6.67	17.56	16.20

Max Sharpe - 2 years		Number of Assets in the portfolio								
Genetic Operator	μ	8	10	12	14	16	18	20	22	24
GM-BM	0.05	22.75	41.10	45.00	31.50	42.11	17.75	38.91	9.88	22.00
GM-BM	0.1	11.09	24.45	36.80	51.90	31.89	17.80	19.80	11.75	11.63
GM-BM	0.15	15.83	20.91	38.90	20.45	46.33	51.25	10.64	8.57	7.00
GM-BM	0.2	11.60	23.55	28.90	23.62	44.40	13.00	35.64	8.14	6.88
GM-BM	0.25	13.45	15.18	21.11	21.73	33.00	28.60	24.50	7.67	9.25
GM-BM	0.3	13.17	18.30	27.78	13.25	37.25	13.50	12.00	7.63	6.25
GM-BM	0.35	12.83	19.73	22.00	24.55	32.78	7.75	12.64	6.75	6.14
GM-BM	0.4	14.45	15.75	30.56	29.00	35.89	16.00	12.36	11.30	6.63
GM-BM	0.45	12.27	16.00	30.00	19.64	40.56	16.20	12.60	7.88	7.13
GM-BM	0.5	13.91	14.83	42.89	34.73	29.50	5.00	18.18	7.00	7.38
GM-BM	0.55	14.18	20.09	36.67	27.64	31.33	13.50	24.55	12.30	7.29
GM-BM	0.6	12.17	16.50	37.88	32.42	63.13	9.33	26.73	6.75	6.00
GM-BM	0.65	15.08	15.90	40.50	16.92	23.44	6.67	23.27	7.25	6.11
GM-BM	0.7	14.82	14.67	40.00	18.92	24.30	13.75	12.64	7.13	5.86
GM-BM	0.75	17.00	17.27	36.00	29.27	61.89	29.00	16.90	6.90	6.57
GM-BM	0.8	10.91	13.67	24.00	62.27	41.44	28.20	37.45	5.89	6.43
GM-BM	0.85	17.45	18.60	29.33	26.31	37.11	32.00	15.64	8.33	6.25
GM-BM	0.9	13.58	19.90	25.00	40.42	19.67	10.80	55.73	6.71	9.00
GM-BM	0.95	17.25	18.17	17.91	19.25	39.11	13.67	55.90	7.25	6.88

MinVol - 1 year		Number of Assets in the portfolio								
Genetic Operator	μ	8	10	12	14	16	18	20	22	24
GM-BM	0.05	79.08	56.38	59.92	73.64	57.77	62.17	108.67	94.55	98.27
GM-BM	0.1	54.50	67.17	54.23	89.23	60.31	75.73	61.08	83.20	80.09
GM-BM	0.15	74.23	71.62	96.31	48.69	92.67	99.00	88.36	84.00	73.45
GM-BM	0.2	78.00	92.58	89.75	79.50	84.08	57.18	78.58	58.82	84.64
GM-BM	0.25	60.46	71.69	89.08	97.15	80.00	89.73	70.75	83.33	108.27
GM-BM	0.3	68.62	98.00	90.92	87.62	109.54	96.73	63.60	84.18	70.91
GM-BM	0.35	55.31	84.58	72.92	60.46	96.82	91.00	72.25	92.82	75.09
GM-BM	0.4	77.50	72.67	79.85	77.46	80.58	84.00	115.42	56.60	81.70
GM-BM	0.45	63.36	82.00	59.00	71.69	78.75	60.89	87.70	68.36	57.00
GM-BM	0.5	80.54	127.38	86.92	89.15	104.42	72.42	63.22	84.36	80.73
GM-BM	0.55	58.33	89.54	67.36	60.23	85.45	76.75	50.55	64.82	81.10
GM-BM	0.6	71.08	89.15	67.08	80.70	71.83	46.50	86.82	78.64	102.45
GM-BM	0.65	73.23	72.54	98.08	79.38	81.91	65.73	62.20	51.70	96.18
GM-BM	0.7	62.85	107.15	92.38	80.23	77.17	86.55	98.78	86.64	70.00
GM-BM	0.75	92.09	80.08	111.08	99.23	113.17	81.75	58.38	71.45	96.82
GM-BM	0.8	70.25	115.69	41.15	76.62	102.58	100.33	117.64	87.64	63.73
GM-BM	0.85	94.92	55.31	88.46	96.23	55.25	51.00	73.27	67.55	77.27
GM-BM	0.9	75.31	74.62	58.69	68.00	71.33	96.42	96.64	111.55	49.10
GM-BM	0.95	84.50	96.62	86.23	80.23	101.58	92.73	106.00	96.80	80.27

MinVol - 2 years		Number of Assets in the portfolio								
Genetic Operator	μ	8	10	12	14	16	18	20	22	24
GM-BM	0.05	45.08	67.62	73.69	50.77	38.91	80.92	60.00	66.91	113.55
GM-BM	0.1	41.38	90.25	93.23	86.50	65.36	89.27	99.70	87.18	71.50
GM-BM	0.15	79.08	66.45	73.67	71.54	69.18	50.08	100.00	93.50	72.10
GM-BM	0.2	90.38	66.67	111.15	63.17	93.00	71.58	56.91	95.00	86.60
GM-BM	0.25	73.92	97.23	60.31	58.15	48.82	110.92	75.82	91.55	88.64
GM-BM	0.3	58.83	94.42	88.00	72.15	87.75	82.22	58.10	82.73	108.36
GM-BM	0.35	104.77	73.75	71.42	73.00	90.64	81.40	74.20	84.80	75.33
GM-BM	0.4	75.75	38.83	73.08	89.00	84.25	47.09	87.09	97.91	75.40
GM-BM	0.45	81.08	74.15	67.17	67.62	85.18	77.36	99.27	93.00	111.50
GM-BM	0.5	97.46	77.33	52.42	59.15	66.42	75.25	90.00	60.70	51.55
GM-BM	0.55	67.54	132.23	87.46	79.92	59.55	109.67	65.70	106.30	85.82
GM-BM	0.6	82.64	63.33	78.67	111.08	67.83	57.36	87.18	90.80	91.27
GM-BM	0.65	69.38	56.69	99.92	76.92	118.67	74.18	58.27	99.60	101.10
GM-BM	0.7	100.46	88.27	62.00	96.38	100.00	71.45	85.64	64.09	79.30
GM-BM	0.75	73.15	105.69	106.00	87.58	67.17	77.33	102.18	66.55	89.40
GM-BM	0.8	95.67	93.50	77.69	80.08	76.58	71.17	78.91	60.55	98.89
GM-BM	0.85	94.38	64.15	69.73	82.25	95.18	83.83	127.70	77.55	60.60
GM-BM	0.9	90.17	60.75	77.42	70.38	54.36	112.58	94.27	98.45	67.91
GM-BM	0.95	94.92	81.58	49.46	73.67	46.08	87.42	90.73	61.40	79.00

Table 4: Average of 100 runs of the number of generations for evolving portfolios of different sizes with different values of μ. Only the GM-μ and BM-μ genetic operators were used in the evolution.

Generation	Genetic operators					
	Flat Crossover	Blend Crossover	Single Point Crossover	BM-μ	GM-μ	Min Max
1	0	0	0	0	1	0
2	1	0	0	0	0	0
3	0	0	1	0	0	0
4	0	1	0	0	0	0
5	0	0	0	1	0	0
6	0	1	0	0	0	0
7	0	0	1	0	0	0
Total	14.29%	28.57%	28.57%	14.29%	14.29%	0.00%

Table 5: Example on how a chromosome was evolved during the life of the EA and which genetic operators were applied. 1 means that the generic operator was applied to the chromosome and 0 means it was not applied.

For the two tasks of maximising the Sharpe ratio and finding the minimum variance portfolio, the BLX-α operator was the most significant operator in finding the solution. This gives an indication that the proportion of the new generations generated by the blend crossover can be increased to improve results further. It can also be concluded, that the impact of the min max operator is negligible and therefore it can be removed from the EA.

4.5 Population size and initial population

The objective of this experiment is to understand the impact of the initial population size on the number of generations required to arrive at the optimum solution, as well as the time required to achieve that. Each simulation was undertaken 100 times, with the results averaged. The simulations were run on portfolios of 20 assets, over a time period ranging from 1 year to 10 years for both optimisation tasks.

The results' pattern is clear for the task of maximising the Sharpe ratio: the lower the population size, the faster (in terms of running time in seconds) the optimum solution can be identified. However, the

Time period in years	Population Size	Number of assets in the portfolio	Genetic operators - values in %					
			Flat Crossover	Blend Crossover	Single Point Crossover	BM-μ	GM-μ	MinMax
1	50	10	15.15	30.08	28.41	17.77	8.01	0.58
2	50	10	14.66	30.28	30.76	17.64	6.35	0.32
5	50	10	15.28	31.98	31.01	16.75	4.82	0.15
10	50	10	11.91	32.91	33.24	18.77	3.16	0.00
1	100	10	11.46	35.95	33.30	12.65	6.30	0.34
2	100	10	9.02	38.52	36.18	11.33	4.62	0.32
5	100	10	11.83	37.43	33.33	12.04	5.02	0.35
10	100	10	11.34	36.71	33.17	17.24	1.53	0.00
1	50	20	7.76	40.68	38.07	11.93	1.56	0.00
2	50	20	7.46	41.63	38.90	10.47	1.54	0.00
5	50	20	7.61	41.32	38.18	11.58	1.31	0.00
10	50	20	7.32	38.83	36.59	16.60	0.66	0.00
1	100	20	8.55	32.91	29.45	20.30	8.75	0.04
2	100	20	8.49	38.34	29.91	17.82	5.43	0.00
5	100	20	9.11	37.06	29.58	18.27	5.87	0.11
10	100	20	7.27	37.92	33.16	18.84	2.82	0.00
1	150	20	7.97	35.65	29.43	19.13	7.81	0.00
2	150	20	8.21	39.39	31.41	15.56	5.43	0.00
5	150	20	8.52	39.23	31.39	16.48	4.38	0.00
10	150	20	6.29	40.64	32.75	17.49	2.82	0.00
1	250	20	7.74	38.47	29.69	16.37	7.73	0.01
2	250	20	7.77	40.84	31.10	15.02	5.24	0.03
5	250	20	7.93	42.93	28.22	14.95	5.95	0.01
10	250	20	6.13	40.88	33.73	15.91	3.35	0.00
1	350	20	6.92	42.65	32.19	13.24	5.01	0.00
2	350	20	7.30	43.18	29.76	14.15	5.60	0.01
5	350	20	7.28	46.44	30.08	13.19	2.99	0.02
10	350	20	5.39	43.01	34.36	14.49	2.74	0.00
1	500	20	6.84	44.09	30.84	12.68	5.55	0.00
2	500	20	7.19	44.26	29.92	13.77	4.85	0.00
5	500	20	7.33	46.38	29.80	13.44	3.05	0.00
10	500	20	5.44	45.76	35.52	11.12	2.16	0.00
1	1000	20	5.22	47.70	32.88	9.89	4.29	0.02
2	1000	20	6.72	48.42	28.78	11.67	4.41	0.00
5	1000	20	5.53	50.39	30.60	10.45	3.02	0.01
10	1000	20	5.74	49.75	34.00	9.61	0.91	0.00

Table 6: Experimental results showing proportion of the used genetic operators in the evolved chromosomes. Results are showing the average out of 100 runs.

larger the population size, the fewer number of generations are required and the longer it takes for each generation to occur.

In considering the minimum variance the pattern is less obvious. The findings in relation to time are consistent: the lower the population size, the faster (in terms of running time in seconds) it is to run the optimum number of generations. However, the initial population size does not give rise to a consistent pattern in the number of generations required.

Optmization task	Number of assets in the portfolio	Number of days	Average Number of Generations							Average Running Time in seconds						
			Population size 50	Population size 100	Populatio n size 150	Population size 250	Population size 350	Population size 500	Population size 1000	Population size 50	Population size 100	Population size 150	Population size 250	Population size 350	Population size 500	Population size 1000
MinVariance	20	252	289.82	74.90	62.22	72.37	84.09	63.20	85.47	2.55	1.24	1.53	3.24	4.88	5.26	14.04
MinVariance	20	504	289.00	88.26	86.44	75.76	78.43	63.41	75.98	4.60	2.72	3.98	5.73	8.30	9.57	281.97
MinVariance	20	1261	283.86	71.76	77.92	65.04	78.06	79.68	87.00	12.54	8.59	10.25	15.13	25.60	1163.56	71.12
MinVariance	20	2519	334.38	87.90	85.35	63.28	82.29	71.10	82.65	27.26	14.15	195.70	25.33	64.23	244.47	1122.36
Max Sharpe	20	252	422.30	382.84	254.42	193.74	100.58	108.06	43.94	3.61	6.49	6.86	332.46	5.79	9.00	7.25
Max Sharpe	20	504	86.96	129.66	94.98	70.78	54.54	45.90	15.64	1.44	4.08	4.52	5.56	5.84	6.96	4.55
Max Sharpe	20	1261	469.72	390.70	345.16	247.62	206.50	138.52	60.30	90.71	45.54	48.43	98.80	69.75	56.55	49.35
MinVariance	10	252	48.57	46.06	48.49	51.96	42.26	47.81	48.46	0.20	0.35	0.53	0.94	1.06	1.73	3.46
MinVariance	10	504	43.34	48.21	41.65	46.06	46.16	47.28	46.93	0.33	0.67	0.84	1.53	2.12	3.20	6.21
MinVariance	10	1261	45.36	48.85	48.10	46.38	48.66	43.07	47.60	0.99	2.02	2.96	4.57	6.71	8.54	18.68
MinVariance	10	2519	47.12	47.74	45.27	44.48	53.76	45.53	40.91	1.86	3.62	4.96	8.05	13.60	16.39	29.95
Max Sharpe	10	252	22.47	23.01	20.63	14.56	15.22	11.72	11.47	0.09	0.17	0.22	0.26	0.38	0.41	0.79
Max Sharpe	10	504	22.07	17.94	14.47	13.72	13.90	11.96	10.02	0.16	0.25	0.29	0.44	0.62	0.75	1.23
Max Sharpe	10	1261	23.28	15.97	17.57	15.07	12.68	11.67	10.26	0.51	0.66	1.07	1.51	1.73	2.22	3.77
Max Sharpe	10	2519	10.71	10.15	9.88	10.45	8.24	7.93	6.53	0.42	0.75	1.06	1.82	1.95	2.74	4.27

Table 7: Average number of generations and average running time for different population size. Results are showing the average of 100 runs. Highlighted in red is the best solution for each portfolio's size in terms of number of generations and in terms of running time.

5 Conclusion

This paper presented an evolutionary algorithm and investigated how it can be utilised to solve two optimisation problems for portfolios with short-selling constraints. It has been shown that evolutionary algorithm can be used to maximise the return of a portfolio while also minimising the risk in a relative short time frame.

The experiments carried out in this paper demonstrate that algorithm's performance can be optimised by choosing the most relevant genetic operators, population size and mutation parameters for the specific data set and the optimisation problem to be solved. This is analogous to tuning hyper-parameters in neural networks.

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